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APPLICATIONS OF MATRIX-ANALYTIC METHODS AND PHASE-TYPE DISTRIBUTIONS IN STOCHASTIC LOGISTIC PROBLEMS MODELING

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Abstract

The goal of this paper is to present a variety of applications of matrix-analytic methods (MAMs) and phase-type (PH) distributions to logistic models with random variables. We first provide an overview of different types of PH distributions as advanced analytic techniques for the solution of non-Markovian state-space based models. In the latter part of the paper, we illustrate these techniques by means of some logistic examples dealing with exponential and non-exponential stochastic processes and random values. The ultimate goal of this paper is to provide a reference for logistic researchers and students in state-space modeling.

***Keywords**: matrix-analytic methods, phase-type distributions, logistic problems, modeling.*

1. INTRODUCTION

Logistic planning and management largely focus on problems of material flows in transportation systems. The very complex processes of transport, within the material flow analysis, are usually represented by simple models in order to find solutions to practical problems. Processes in real material flow systems, the movement of transport units or changes in inventories at warehouses, can be modeled as changes in characteristic system values over time. These system values can be interpreted as random variables and their changes over time as stochastic processes. In this sense, discrete and continuous probability distributions are the basis for analytical study and simulation of transportation processes with dynamic behavior.

The most important random variables in a material flow analysis, as a major category of logistics processes, are:

- results in time intervals, for example number of transportation orders, number of operating cycles for a period, total number of failures, number of transportation units necessary for the movement of the product, number of arrivals per unit time, etc.,
- time interval between events, for example interarrival time, sojourn time, service time, waiting time in front of server, time gaps, lead time and cycle time, etc.,
- distance traveled during a time interval, for example length of road of FTS vehicles or trucks in external transport, length of transportation networks, etc.,
- the amount of material to be handling in a given time interval, for example in the manufacture and storage of goods in distribution centers of logistics chains, etc [1].

In general, random variable can be defined as a numerical outcome that results from an experiment. Discrete random variable can take on only a finite or countably infinite set of outcomes (the first group of the above examples - total number of failures). On the other hand, continuous random variable can take on any value along a continuum or infinite set of outcomes (the second group of the above examples - interarrival time).

In view of the above presented facts that changes in characteristic system values over time have stochastic character, effective managing of logistics processes, particularly in terms of optimization, requires the involvement of probability theory and stochastic process, as well as reliability theory. On this basis, it is possible to define appropriate mathematical models that would adequately interpret complex logistic processes and that are very important for optimal system management.

2. LOGISTIC PROCESS WITH NON-EXPONENTIAL BEHAVIOR

In modeling different logistic processes the basic idea is to graphically describe the real-time dynamical evens or flows by state-space diagrams that may be further transformed into mathematical models for which solution mathematical tools and procedures are well known. A standard form for state-space diagram is directed graph or digraph **G**(**V**,**E**) composed of the following elements **V** – set of nodes (vertex) and **E** – set of edges (links between vertices). Nodes represents the different states of the domain (i.e. cities in the transportation problem) and edges represents the transitions from a state to another. Each edge is a pair (*i*,*j*), where *i* and *j* belongs to V. If the edge pair is ordered, the edge is called directed and thus the graph is directed graph. Otherwise, the graph is called undirected and it's rarely encountered in usual logistical problems. Very often, an edge has a component called edge cost (or weight).

It is generally known that the state-space diagram can be simply represented by *Markov processes* (a kind of a stochastic processes) if transition probabilities, from a state to another, have constant values (do not depend on time) and if future states of the system depend only on the present state (not on any past states). This means that the time spent in a state *i*, before the system moves to the next state *j*, takes non-negative real values and has an exponential distribution which further defines transition probability, from state *i* to state *j* as:

$$p_{ij} = \frac{1}{E(t)} = const., \quad (1)$$

where $E(t)$ denote expected value of the time spent in state i.
If it's necessary to model random variables (processes) which are characterized by general distribution (non-exponential) functions, such models are called non-Markovian. This class of models can be solved using several different approaches:

- Markov renewal theory [2]
  - Markov renewal sequences,
  - semi-Markov processes
  - Markov regenerative processes,
- method of additional variables [3],
- matrix-analytic methods (MAMs) and their part phase-type (PH) distributions [4].

The most important advantage of using PH distributions is their mathematical tractability, which is primarily reflected in the possibility of approximation of arbitrary continuous probability distributions with arbitrary precision. Namely, an increase in the number of phase (stages) causes an increase in precision of approximation. In contrast, the application of additional variables method or Markov renewal theory is very limited in practical problems [5].
The fact that some general distribution or an empirical data set can be approximated by two or more exponential distributions is very often used in logistic models where transition processes have non-exponential behaviour.

3. MOSTLY USED CONTINUOUS PHASE-TYPE DISTRIBUTIONS

PH distributions are based on the method of stages technique introduced by A. K. Erlang (1917.) [6] and later (1981) generalized by M. F. Neuts [4]. Since their introduction PH distributions have been used in a wide range of stochastic modelling applications in different areas such as: telecommunications, finance, biostatistics, queueing theory, reliability theory, survival analysis, etc [7]. Neuts defined PH distribution as the distribution of the time until absorption in a Markov process with a finite number n of transient states and one absorbing state, state n+1 [4]. The key idea is to model random time intervals (with non-exponential distribution) as being made up of a number of exponentially distributed segments and to exploit the resulting Markovian structure to simplify the analysis.
Let $X(t)$, $t \geq 0$, be a time-homogeneous Markov process with discrete state space $\{1, ..., n, n+1\}$ and infinitesimal generator (hereinafter only generator) Λ:

$$\mathbf{\Lambda} = \begin{pmatrix} \mathbf{\Theta} & \boldsymbol{\theta} \\ \mathbf{0} & 0 \end{pmatrix}, \quad (2)$$

where $\mathbf{\Theta}$ is a $n \times n$ square matrix (generator restricted to the transient states), $\boldsymbol{\theta}$ column vector and $\mathbf{0}$ row vector of order n. The initial probability vector of process $X(t)$ is denoted by $\acute{\boldsymbol{\alpha}} = (\boldsymbol{\alpha}, \alpha_{n+1})$ where $\boldsymbol{\alpha}$ is a row vector of size n. The states $\{1, ..., n\}$ are referred to the transient states and n+1 is an absorbing state. Let Z:= inf $\{t \geq 0 : X(t) = n+1\}$ be the time until absorption of the process $X(t)$ in state n+1. The distribution of Z is called phase-type (PH) distribution with parameters $\boldsymbol{\alpha}$ and $\mathbf{\Theta}$ and is denoted by PH($\boldsymbol{\alpha}$,$\mathbf{\Theta}$). Dimension n of matrix $\mathbf{\Theta}$ is order of PH distribution and represents the number of phases or stages. The basic distributional characteristics of PH distribution (the cumulative distribution function (3), the density function (4) and the rth moment (5)) are:

$$F(t) := \mathrm{P}(Z \leq t) = 1 - \boldsymbol{\alpha} e^{\mathbf{\Theta} t} \mathbf{e}, \quad (3)$$

$$f(t) = \boldsymbol{\alpha} e^{\mathbf{\Theta} t} \boldsymbol{\theta}, \quad (4)$$

$$m_r = (-1)^r r! \boldsymbol{\alpha} \mathbf{\Theta}^{-r} \mathbf{e}, \quad (5)$$

where $e = (1,...,1)^{\mathrm{T}}$ is a column vector of ones and r is ordinal number of a moment.
According to form of matrix $\mathbf{\Theta}$ and initial probability vector $\boldsymbol{\alpha}$ it is possible to classify different types of PH distributions: exponential distribution (one phase PH distribution), Erlang distribution (two or more identical phases in sequence), hypoexponential distribution (two or more non necessarily identical phases in sequence – series connection), hyperexponential distribution (two or more non necessarily identical phases - parallel connection), Coxian distribution (two or more not necessarily identical phases in sequence, but with a probability of transitioning to the absorbing state after each phase), etc.
The basic indicator in selecting one of these distributions to represent a non-exponential distribution is the coefficient of variation. The coefficient of variation CV is a measure of deviation from the exponential distribution (CV = 1) [5]. Table 1 shows the intervals of the coefficient of variation for some types of PH distributions.

Table 1 Koefficient of variation for some types of PH distributions

coefficient of variation CV	Type of PH distribution
> 1	hyperexponential
1	exponential
< 1	hypoexponential
0	deterministic distribution

PH distributions capture a wide range of statistical characteristics including high variability. Note that PH distributions do not capture long-range dependence or self similar behavior. There is another set of processes known as Markovian arrival processes that are still based on the method of stages and capture long-range dependence in a data set. PH distributions are a special case of Markovian arrival processes [8].
The following probability distributions are considered as special cases of a continuous PH distribution. Moreover, each of them has been used widely in literatures.

3.1 Exponential distribution

Exponential distribution is one of the most important continuous theoretical distribution which describe many natural phenomena. The density function of exponential distribution is:

$$f(t) = \begin{cases} \lambda \cdot e^{-\lambda \cdot t} & \text{za} \quad 0 \leq t \leq \infty \\ 0 & \text{za} \quad t < 0 \end{cases}, \quad (6)$$

where parameter λ determines the "rate" at which events occur. The cumulative distribution function is defined as:

$$F(t) = \begin{cases} 1 - e^{-\lambda \cdot t} & \text{za} \quad 0 \le t \le \infty \\ 0 & \text{za} \quad t < 0 \end{cases} \tag{7}$$

In general, any exponentially distributed random variable $t \sim \text{Exp}(\lambda)$, with parameter λ, has the following properties:

- expected value $E(t) = 1/\lambda$,
- moment about zero $m_r = r!/\lambda^r$,
- variance $Var(t) = 1/\lambda^2$,
- coefficient of variation CV = 1,
- skewness $\alpha 3 = 2$,
- kurtosis $\alpha 4 = 6$,
- transient generator $\Theta = [-\lambda]$.

It is easy to observe that an exponential distribution is also a phase-type distribution which has only one phase. Consequently, processing time till the absorbing state is just moving from initial state to the absorbing state.

Exponential distributions dominant feature is "ease-to-use" character in practical engineering situations. Applying the exponential distribution is relative simply in stochastic modeling because there is only one parameter λ. The great significance of this distribution is in the fact that it is unique continuous theoretical distribution with so called memory less property. The memory less property enables simple expressions for many performance measures of stochastic logistic models. The third important feature of exponential distribution is its relation to the Poisson distribution. This distribution is used to measure the time intervals between events according to Poisson process.

Exponential distribution has many important features that often provide analytical solutions of the problem. On the other hand, it is not always the ideal approximation of the observed phenomena in nature. Coefficients of variation of many important processes and random variables have values which are significantly more or less than one. This means that it is necessary to define some other PH distributions which can be better approximation of non-exponential processes.

3.2 Hypo - exponential distribution

Hypoexponential distribution or generalized Erlang distribution is the probability distribution of time to absorption in Markov process with two or more non necessarily identical, series-connected, exponentially distributed phases (states). The continuous, non-negative random variable $t \sim \text{Hypo}(k, \lambda_i)$ has hypoexponential distribution if its density function has a form:

$$f(t) = \sum_{i=1}^{k} \prod_{i \neq j} \frac{\lambda_j}{\lambda_j - \lambda_i} \cdot \lambda_i \cdot e^{-\lambda_i \cdot t}, \tag{8}$$

where k is the number of phases and λ_i is transition rate from the i-th phase. The cumulative distribution function is defined as:

$$F(t) = \sum_{i=1}^{k} \prod_{i \neq j} \frac{\lambda_j}{\lambda_j - \lambda_i} \cdot e^{-\lambda_i \cdot t}. \tag{9}$$

Figure 1 shows a state transition diagram – graph of hypoexponential distribution.

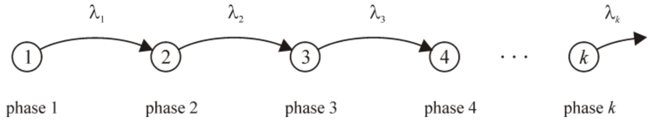

Fig. 1 State transition diagram of hypo - distribution

Random variable $t \sim \text{Hypo}(k, \lambda_i)$ has the following properties:

- expected value $E(X) = \sum_{i=1}^{k} 1/\lambda_i$,
- moment about zero do not exist in closed form
- variance $Var(X) = \sum_{i=1}^{k} 1/\lambda_i^2$,
- coefficient of variation CV < 1,
- skewness $\alpha_3 = \dfrac{2 \cdot \sum_{i=1}^{k} \frac{1}{\lambda_i^3}}{(\sum_{i=1}^{k} \frac{1}{\lambda_i^2})^{3/2}}$,
- kurtosis do not exist in closed form
- transient generator

$$\Theta = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 & \dots & 0 & 0 \\ 0 & -\lambda_2 & \lambda_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -\lambda_{k-2} & \lambda_{k-2} & 0 \\ 0 & 0 & \dots & 0 & -\lambda_{k-1} & \lambda_{k-1} \\ 0 & 0 & \dots & 0 & 0 & -\lambda_k \end{bmatrix}.$$

- initial probability vector $\alpha = (1, 0, ..., 0)$.

While the Erlang distribution is a series of k exponential distributed phases all with rate λ, the hypoexponential is a series of k exponential distributions each with their own rate λ_i.

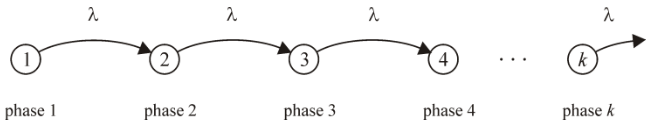

Fig. 2 State transition diagram of Erlang distribution

As a result of that, Erlang distribution can be considered as a special case of the hypoexponential distribution.

3.3 Hyper - exponential distribution

Hiperexponential distribution is the probability distribution of time from initial state to absorption in Markov process with two or more non necessarily identical, parallel-connected, mutually exclusive, exponentially distributed phases (states). The continuous, non-negative random variable $t \sim \text{Hyper}(k, \alpha_i, \lambda_i)$ is distributed according to hyperexponential distribution if its density function is defined as:

$$f(t) = \sum_{i=1}^{k} \alpha_i \cdot \lambda_i \cdot e^{-\lambda_i \cdot t}, \tag{10}$$

where k is the number of phases, λ_i is transition rate from the i-th phase and α_i is probability of transition to the i-th phase (component of initial probability vector). The cumulative distribution function is defined as:

$$F(t) = 1 - \sum_{i=1}^{k} \alpha_i \cdot e^{-\lambda_i \cdot t} \,. \tag{11}$$

Figure 3 shows a state transition diagram – graph of hyperexponential distribution.

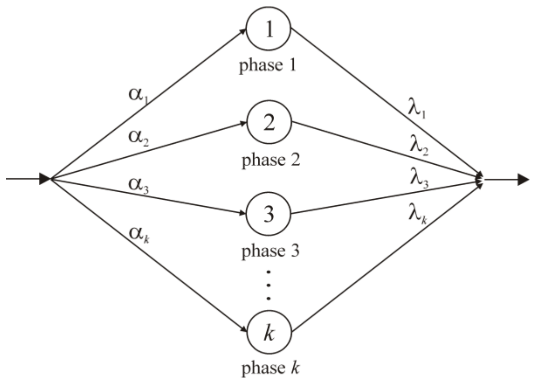

Fig. 3 State transition diagram of hyper - distribution

Random variable t~Hyper(k, α_i, λ_i) has the following properties:

- expected value $E(X) = \sum_{i=1}^{k} \alpha_i / \lambda_i$,
- moment about zero $m_r = r! \cdot \sum_{i=1}^{k} \alpha_i / \lambda_i^r$,
- variance $Var(X) = \sum_{i=1}^{k} \frac{2\alpha_i - \alpha_i^2}{\lambda_i^2}$,
- coefficient of variation CV > 1,
- skewness do not exist in closed form
- kurtosis do not exist in closed form
- transient generator

$$\mathbf{\Theta} = \begin{bmatrix} -\lambda_1 & 0 & ... & 0 \\ 0 & -\lambda_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & -\lambda_k \end{bmatrix},$$

- initial probability vector $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_k)$, . of order k where $\sum_{i=1}^{k} \alpha_i = 1$.

The hyperexponential distribution exhibits more variability than the exponential (CV > 1). Typical examples of application are CPU service-time distribution in a computer system and the failure density of a product manufactured in several parallel assembly lines which outputs are merged.

4. MATRIX-ANALYTIC METHODS FOR MODELING GENERALLY DISTRIBUTED TIMES IN LOGISTIC SYSTEMS

Recent applications of matrix-analytic methods in queueing theory, reliability and availability, telecommunications, civil engineering, finance, computer science [9], among others, shows the power of these techniques in different areas. In this paper we focus on the applications of matrix-analytic methods in two very importante areas and we present the concepts and the modeling approach of real-life logistics problems.

4.1 Matrix-analytic methods in queuing theory

The first example is model described in the paper *Application of the Markov theory to queuing networks* by Petrovic et al. [10]. This paper presents an application of the matrix-analytic methods to the model of networked transport system which consists of two subsystems, namely PS1 i PS2 (Fig. 4). Transport units (TU) enter subsystem PS1 and are processed. In part they depart from the system, while partly, they come to the second subsystem PS2. At the entrance of the subsystem PS2 the units coming from the outside are included too. After being processed in subsystem 2, the units depart from the system. The aim is to determine average number of transport units in each subsystem as well as average time of keeping the unit within each subsystem.

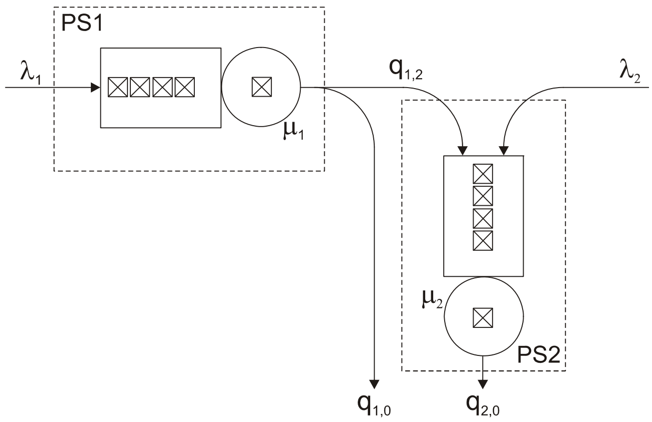

Fig. 4 Model of Real Transport System

Number of transport units entering subsystems PS1 is modeled by Poisson's distribution with parameter λ_1, while the TU processing time in subsystem PS1 is defined by exponential distribution with parameter μ_1. Number of TUs departing from the system is defined by parameter $q_{1,0}$, while the number of units entering into queue of subsystem PS2 is defined by parameter $q_{1,2}$. The queue of subsystem PS2 also includes the units coming from the environment by Poisson's distribution with parameter λ_2. TUs leave subsystem PS2 after the processing which is defined by exponential distribution of service time with parameter μ_2. Thus defined numerical example represents an open network of the queuing system for whose modeling methodology described in previous sections is applied.

The set model represented in the form of the graph of states is given in Fig. 5. If the capacity of both subsystems PS1 and PS2 are S_1=S_2=5 then the transient generator matrix gets the form shown by expression 13, where -Σi represents negative sum of all the elements in i-th row. Also, initial probability vector has a form:

$$\boldsymbol{\alpha} = (1, 0, ..., 0). \tag{12}$$

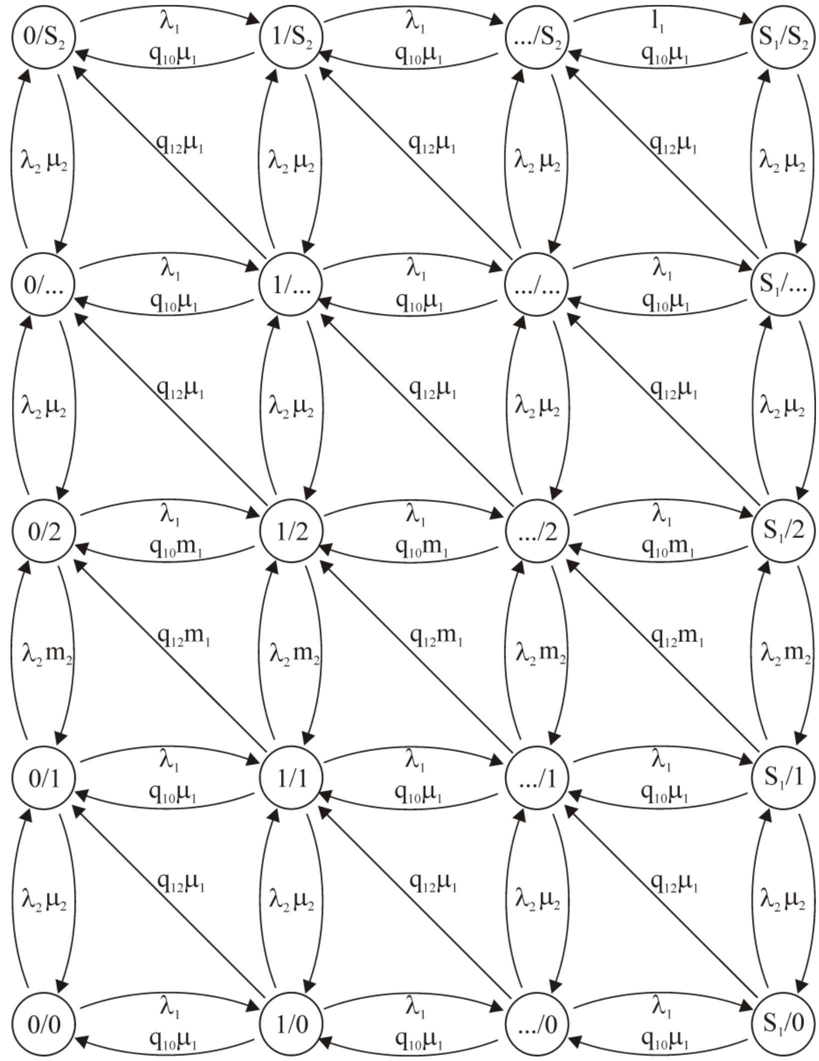

Fig. 5 Model represented in the form of the graph

$$
\Theta = \begin{array}{c|cccccccccccccccc|}
 & (0,0) & (1,0) & (2,0) & (3,0) & (4,0) & (5,0) & (0,1) & (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (0,2) & (1,2) & \cdots & (4,5) & (5,5) \\
(0,0) & -\Sigma_1 & \lambda_1 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\
(1,0) & q_{10}\mu_1 & -\Sigma_2 & \lambda_1 & 0 & 0 & 0 & q_{12}\mu_1 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\
(2,0) & 0 & q_{10}\mu_1 & -\Sigma_3 & \lambda_1 & 0 & 0 & 0 & q_{12}\mu_1 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\
(3,0) & 0 & 0 & q_{10}\mu_1 & -\Sigma_4 & \lambda_1 & 0 & 0 & 0 & q_{12}\mu_1 & \lambda_2 & 0 & 0 & 0 & 0 & & 0 & 0 \\
(4,0) & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_5 & \lambda_1 & 0 & 0 & 0 & q_{12}\mu_1 & \lambda_2 & 0 & 0 & 0 & & 0 & 0 \\
(5,0) & 0 & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_6 & 0 & 0 & 0 & 0 & q_{12}\mu_1 & \lambda_2 & 0 & 0 & & 0 & 0 \\
(0,1) & \mu_2 & 0 & 0 & 0 & 0 & 0 & -\Sigma_7 & \lambda_1 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & & 0 & 0 \\
(1,1) & 0 & \mu_2 & 0 & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_8 & \lambda_1 & 0 & 0 & 0 & q_{12}\mu_1 & \lambda_2 & \cdots & 0 & 0 \\
(2,1) & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_9 & \lambda_1 & 0 & 0 & 0 & q_{12}\mu_1 & & 0 & 0 \\
(3,1) & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_{10} & \lambda_1 & 0 & 0 & 0 & & 0 & 0 \\
(4,1) & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_{11} & \lambda_1 & 0 & 0 & & 0 & 0 \\
(5,1) & 0 & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_{12} & 0 & 0 & & 0 & 0 \\
(0,2) & 0 & 0 & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{13} & \lambda_1 & & 0 & 0 \\
(1,2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & q_{10}\mu_1 & -\Sigma_{14} & & 0 & 0 \\
\cdots & & & & & & & & & \cdots & & & & & & & & \\
(4,5) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & -\Sigma_{35} & \lambda_1 \\
(5,5) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & q_{10}\mu_1 & -\Sigma_{36}
\end{array} \qquad (13)
$$

4.2 Phase-Type Distribution for modeling failure rate function

The problem analized in reference [9] is shown here as the second example of matrix-analytic methods in logistics. In order to study system performance throughout its life cycle, method of multi – state degradation analysis was introduced [11]. Degradation is a continuous random process in time, and generally, it can be modeled by a continuous probabilistic function. However, in practice, the description of the system operating (technical) condition is accomplished through a finite number of system states, and hence, the continuous degradation path is simplified by dividing it into a number of different discrete states [12].

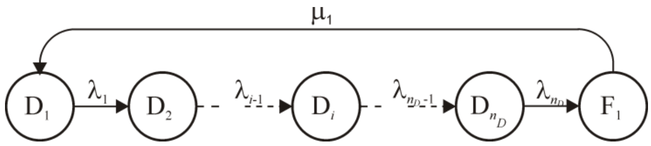

Fig. 6 System degradation path:
D_i – degradation states, F_1 – degradation failure

Degradation state D_i (degradation level or cluster) can be introduced as a system state with relevant technical conditions at similar level of operating ability. Because of that, failure rate function has close relationship with degradation process and in a specific degradation level, system failure rate is assumed to have a constant value λ_i, i=1÷n_D. The final area (degradation level n_D) represents the state of the system with a strong increase in failure rate function (approximated with value λ_{nD}) when the degradation failure F_1 occurs. Time to degradation failure F_1, in mathematical sense, represents the generalized Erlang or hypoexponential distribution as a special case of PH distribution. The last process necessary to complete the model is replacement/repair of failed system. After degradation failure the system will be replaced or repaired to a state D_1 which is "as good as new" state. Time of replacement or repair is exponentially distributed E(μ_1). If corrective maintenance time, after degradation failure, is not exponentially distributed, process can be modeled (similarly to degradation process) as another hypoexponential distribution. The aim is to determine working state probabilities (availability) as well as probability of failure state.

The transient generator matrix (Θ) and initial probability vector (α), for system degradation model through n_D states, with degradation failure and corective maintenance can be represented expressions (14) and (15) respectively:

$$\Theta = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_2 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \cdots & & & & & \\ & & & \cdots & & & & & \\ 0 & 0 & 0 & & -\lambda_i & \lambda_i & 0 & 0 & 0 \\ & & & & & & \cdots & & \\ & & & & & & \cdots & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{n_D} & \lambda_{n_D} \\ \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 \end{bmatrix}, \quad (14)$$

$$\alpha = (1, 0, ..., 0). \quad (15)$$

5. CONCLUSION

Markov models are a well known modeling technique in industrial and academic applications. Aim of this paper was to present applications of matrix-analytic methods and phase-type distributions to logistic models with random variables.

The presented methodology enables a rapid increase in the size of the problems that can be effectively handled by Markov models. It offers a new possibility of dealing with non-exponential processes and variables. Matrix-analytic methods and phase-type distributions represents flexible and effective modeling method and the author's intent is that this paper will be an encouragement to logistic researchers and students in state-space modeling.

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